PG – 967

I Semester M.Sc. Examination, January 2015
(Y2K11 (RNS) Scheme)
MATHEMATICS
M103 : Topology – I

Time : 3 Hours   Max. Marks : 80

Instructions : *1) Answer **any five full** questions choosing atleast **two** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) Define an infinite set. Prove that every super set of an infinite set is infinite. **8**

   b) Let X be an infinite set, and $x_0 \in X$ then prove that $X - \{x_0\}$ is an infinite set. **8**

2. a) Prove that the open interval (0, 1) of reals is non-denumerable set. **8**

   b) State and prove Cantor's theorem. **8**

3. a) Define a metric on a nonempty set X. If d is a metric on X, then prove that

   $e(x, y) = \frac{d(x,y)}{1+d(x,y)}$, $\forall x, y \in X$ is a metric on X. **8**

   b) Prove that a subspace of a complete metric is complete iff it is closed. **8**

4. a) State and prove contraction mapping theorem. **6**

   b) State and prove Baire's category theorem. **10**

PART – B

5. a) Define a topology on a non-empty set. Prove that the intersection of two topologies is again a topology. **5**

   b) Is the union of two topologies a topology ? Justify. **3**

   c) Prove that every metric space is a topological space. **8**

P.T.O.

6. a) Prove the following hold in [x, $\Im$]. **6**

i) $\alpha(\phi) = \phi$

ii) $A \subseteq B$ implies $d(A) \subseteq d(B)$

ii) $d(A \cup B) = d(A) \cup d(B)$

b) If $A \subseteq (X, \Im]$, then prove that $A \cup d(A)$ is closed. **4**

c) Prove that a point x belongs to the closure of a set A iff every open set G which contains x has a nonempty intersection with A. **6**

7. a) Prove the following :

i) $A^\circ \subseteq A$

ii) A is open iff $A = A^\circ$

iii) $A \subseteq B \Rightarrow A^\circ \subseteq B^\circ$

iv) $A^\circ \cap B^\circ = (A \cap B)^\circ$ **8**

b) Let $(Y, \Im^*) \subseteq (X, \Im)$ and $E \subseteq Y \subseteq X$, then prove the following

i) $\overline{E}_y = \overline{E} \cap Y$

ii) $E^\circ = E^\circ{}_Y \cap_Y {}^\circ$

iii) $b_Y(E) \subseteq b(E) \cap Y$ **8**

8. a) Prove that a mapping $f : X \rightarrow Y$ is continuous iff inverses of open sets are open. **8**

b) Prove that a bijective function $f : X \rightarrow Y$ is a homeomorphism iff

$f(A^\circ) = f(A)^\circ \;\; \forall\, A \subseteq X$. **8**